Information Statistics II

Lecture 9. Threshold decomposition and logical filter family

Threshold decomposition

Threshold decompotion is a property that a filtering procedure on a grayscale image is decomposed into a set of the procedures on a set of binary images produced by thresholding the original grayscale image. The median filter has this property.

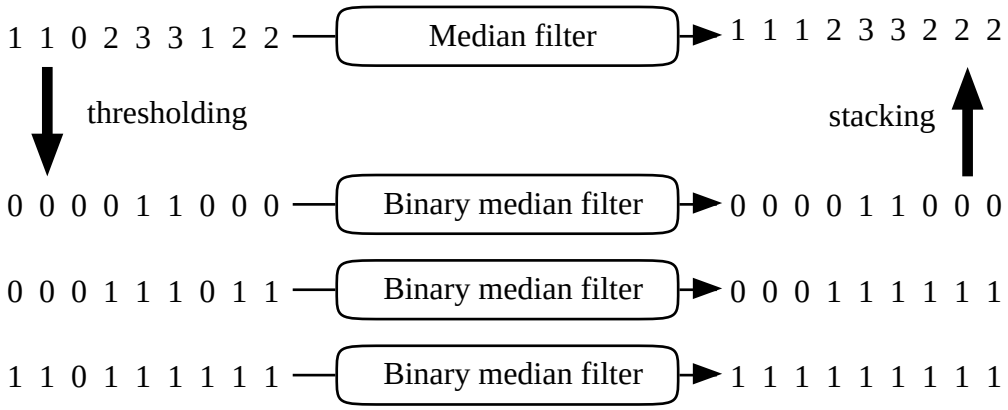

Fig. 1. Threshold decomposition.

Intuitive proof of the threshold decomposition property

In Fig. 2 all pixel values in the filter window at a position are displayed by thick vertical lines in incremental order. The horizontal axis indicates the rank of each pixel value in the window; the vertical one indicates pixel values. Each horizontal line indicates a thresholding level. At the level I the median is 0 since less than half of the binarized pixel values are 1. At the level III the median is 1 since more than half of the values are 1. The boundary between the levels where the median is 1 and 0 is the level II. At this level, half of the binarized pixel values are 1 and half are 0. The level equals to the grayscale median value, since the numbers of the pixel values greater than and less than this value are the same.

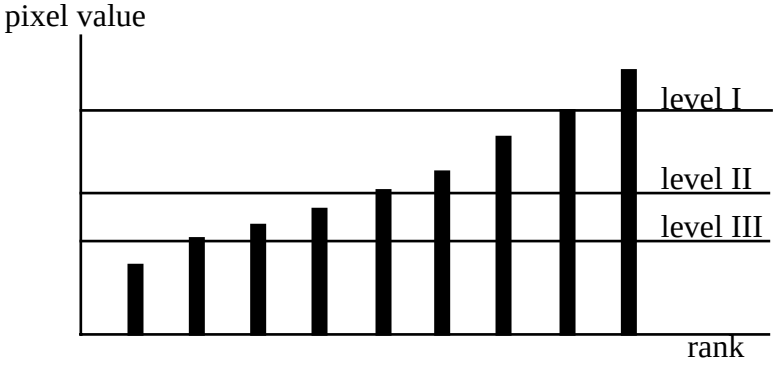

Fig. 2. Sorted pixel values in a window.

Stacking property

At the set of binary images produced by thresholding, if the value at a position is 1 in a binary image corresponding to a graylevel, the values at this position are always 1 in binary images corresponding to graylevels lower than that graylevel.

A filter has the stacking property if this ordering is preserved by a filtering. If a filter has the stacking property, the reconstruction of the grayscale filter output is achieved only by finding the boundary of 0 and 1 of the binary outputs at each position. The stacking property is naturally expected for noise-removing filters to have, and is equivalent to the increasing property in the morphological sense.

Stack filters

The stack filters are defined by binary operations on each binary image produced by thresholding. The binary operation is defined as a logical function of pixels in a window on binarized images. The logical functions of stack filters are restricted to those which hold the stacking property. Such class of logical functions is known as the positive function.

Weighted median filter and neural filters

The weighted median filter assigns a weight coefficient to each position in a window. The filter output is the median of the sequence of pixel values, each of which appears duplicatedly. If weight coefficient is *n* at a position, the value at this position appears *n* times.

The weighted median filter has also the threshold decomposition property. In case of binary images, the weighted median operation is reduced to weighted summation followed by thresholding. This class of operations is called the threshold logic, and is a subset of the class of positive functions.

This property suggests that the weighted median filter is expressed by a configuration of the artificial neural network. The neural filters are extension of the weighted median filter and defined by the neural network itself.

Reference

J. P. Fitch, E. J. Coyle and N. C. Gallagher, Jr., "Median Filtering by Threshold Decomposition," *IEEE Trans. Acoust., Speech, Signal Processing,* **ASSP-32,** 6, 1183-1188 (1984).

P. D. Wendt, E. J. Coyle and N. C. Gallagher, Jr., "Stack Filters," *IEEE Trans. Acoust., Speech, Signal Processing,* **ASSP-34,** 4, 898-911 (1986).

weight coefficients

1	*1*	*1*
1	*3*	*1*
1	*1*	*1*

binary pixel values

0	0	1
0	1	1
0	0	0

Output: median(0,0,1,0,**1,1,1,**1,0,0,0) = 0

Equivalent to:
1. Summation 0+0+1+0+**3*1**+1+0+0+0 = 5
2. Thresholding 5 < (sum of weight coefficients / 2) : output 0

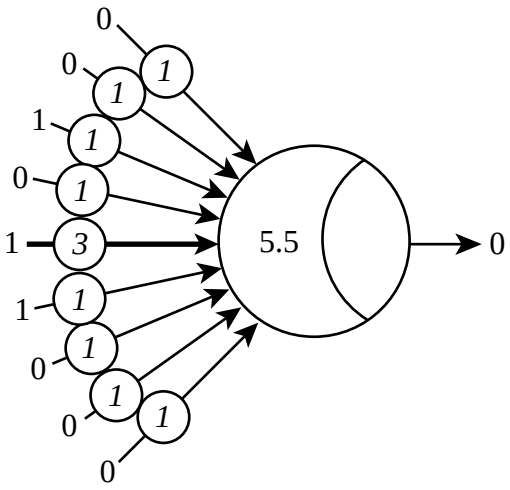

Fig. 3. Equivalence of binary weighted median filter and artificial neuron